JEE ADVANCED PAPER – 1

APNI KAKSHA RANKERS PROGRAM

Time : 3 Hours **Maximum Marks : 180**

ANSWER KEY
&
SOLUTION WITH EXPLANATION

Solution Sheet Available Only for 24 Hours

PART – 1 : MATHEMATICS

SECTION 1

- This section contains **FOUR (04)** questions having four options each.
- **ONLY ONE** of these four options is the correct answer.

Full Marks	: **+3**	If ONLY the correct option is chosen;
Zero Marks	: **0**	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: **−1**	In all other cases.

1. If $\frac{2}{1!9!}+\frac{2}{3!7!}+\frac{1}{5!5!}=\frac{2^m}{n!}$ then orthocentre of the triangle having sides $x-y+1=0, x+y+3=0$ and $2x+5y-2=0$ is

(A) $(2m-2n, m-n)$ (B) $(2m-2n, n-m)$

(C) $(2m-n, m+n)$ (D) $(2m-n, m-n)$

Ans. **(A)**

Sol. $\because \frac{2}{1!9!}+\frac{2}{3!7!}+\frac{1}{5!5!}=\frac{2^m}{n!}$

$\frac{1}{10!}\left\{\frac{(2)10!}{1!9!}+\frac{(2)10!}{3!7!}+\frac{10!}{5!5!}\right\}=\frac{2^m}{n!}\Rightarrow\frac{1}{10!}\{2\,{}^{10}C_1+2\,{}^{10}C_3+{}^{10}C_5\}=\frac{2^m}{n!}$

$\frac{1}{10!}\{{}^{10}C_1+{}^{10}C_3+{}^{10}C_5+{}^{10}C_7+{}^{10}C_9\}=\frac{2^m}{n!}\Rightarrow\frac{1}{10!}(2)^{10-1}=\frac{2^m}{n!}\therefore m=9$ and $n=10$

Hence $x-y+1=0$ and $x+y+3=0$ are perpendicular to each other then orthocentre is the point of intersection which is $(-2,-1)$

$\therefore -2=2m-2n$ and $-1=m-n$.

2. If the circles $z\bar{z}+\bar{a}z+a\bar{z}+b=0$ and $z\bar{z}+\bar{c}z+c\bar{z}+d=0$, b and d are real, cut orthogonally, then Re $(a\bar{c})$ equal to

(A) $b+d$ (B) $b-d$

(C) $\frac{b+d}{2}$ (D) $\frac{|b+d|}{2}$

Ans. **(C)**

Sol. centre radius

$-a$ $\sqrt{|a|^2 - b}$

$-c$ $\sqrt{|c|^2 - d}$

For orthogonal

$r_1^2 + r_2^2 = d^2$

$|a|^2 - b + |c|^2 - d = |a - c|^2$

$-b - d = -a\bar{c} - \bar{a}c$

$\text{Re}\,(a\bar{c}) = \frac{b+d}{2}$

3. Let $f: D \to y$. If $f(x) = \ln\left[\cos\ |x| + \frac{1}{2}\right]$, (where $[\cdot]$ represents G.I.F), then $\int_{x_1}^{x_2}\left(\text{Lt}_{n\to\infty}\left(\frac{(f(x))^n}{x^2+\tan^2\ x}\right)\right)dx$ where $(x_1, x_2 \in D)$ is

(A) 0 (B) 1

(C) $\frac{3}{2}$ (D) 2

Ans. **(A)**

Sol. If $\frac{1}{2} \le \cos\ |x| \le 1$

$f(x) = \ln\ 1 = 0$

For $\cos\ |x| < \frac{1}{2}$

$f(x)$ is not defined.

4. If $(a - a')^2 + (b - b')^2 + (c - c')^2 = p$ and $(ab' - a'b)^2 + (bc' - b'c)^2 + (ca' - c'a)^2 = q$, then the perpendicular distance of the line $ax + by + cz = 1, a'x + b'y + c'z = 1$ from origin, is

(A) $\sqrt{\frac{p}{q}}$ (B) $\sqrt{\frac{q}{p}}$ (C) $\frac{p}{\sqrt{q}}$ (D) $\frac{q}{\sqrt{p}}$

Ans. **(A)**

Sol. Any plane containing the line is $p_1 + \lambda p_2 = 0$. It's distance (d) from origin $= \frac{|1+\lambda|}{\sqrt{\Sigma(a+\lambda a')^2}}$.

Let $d^2 = \phi = \frac{\lambda^2+2\lambda+1}{A\lambda^2+B\lambda+C}$ where $A = \Sigma a'^2, B = 2\Sigma aa'$ and $C = \Sigma a^2$.

$\Rightarrow\ (A\phi - 1)\lambda^2 + (B\phi - 2)1 + (C\phi - 1) = 0$

As $\lambda \in R, D \ge 0 \Rightarrow\ 0 \le \phi \le \frac{4(A+C-B)}{4AC-B^2} = \frac{p}{q}$ So, $d_{max} = \sqrt{\frac{p}{q}}$

SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.

Full Marks : **+2** If ONLY the correct numerical value is entered at the designated place;
Zero Marks : **0** In all other cases.

Question Stem-1

Let $f(x) = \sin^3 x + 8\cos^6 x - 3, g(x) = 2 + 3\sin 2x\cos x$ and $h(x) = \sin x + 3\cos 2x\sin x$.

5. If $g(x) + h(x) = 0$, then number of solution in the interval $x \in [0,2\pi]$, is

Ans. **(5)**

Sol. $g(x) + h(x) = 0$

$\Rightarrow 2 + 3\sin 2x\cos x + \sin x + 3\cos 2x\sin x = 0$

$\Rightarrow 2 + 3\sin 3x + \sin x = 0$

$\Rightarrow 2 + 9\sin x - 12\sin^3 x + \sin x = 0$

$\Rightarrow 12\sin^3 x - 10\sin x - 2 = 0$

$\Rightarrow (\sin x - 1)(12\sin^2 x + 12\sin x + 2) = 0$

$\Rightarrow \sin x = 1 \text{ or } \frac{-12+\sqrt{48}}{24} \text{ or } \frac{-12-\sqrt{48}}{24}$

$\therefore$ Total 5 solutions.

6. If $f(x) + g(x) = 0$, then number of solutions in the interval $x \in \left[\frac{-\pi}{2}, \frac{5\pi}{2}\right]$ is

Ans. **(5)**

Sol. $f(x) + g(x) = 0$

$\Rightarrow \sin {}^3x + 8\cos^6 x - 3 + 2 + 3\sin 2x\cos x = 0$

$\Rightarrow \sin {}^3x + (2\cos^2 x)^3 + (-1)^3 = 3(\sin x)(2\cos^2 x)(-1)$

$\Rightarrow \sin x + 2\cos^2 x - 1 = 0$

or $\sin x = 2\cos^2 x = -1(\text{ reject })$

$\Rightarrow \sin x + 2(1 - \sin^2 x) - 1 = 0 \quad \Rightarrow 2\sin^2 x - \sin_{x-1} x - 0$

$\Rightarrow \sin x = \frac{-1}{2} \text{ or } 1 \Rightarrow x = \frac{-\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{2} \Rightarrow 5 \text{ solutions}$

Question Stem-2

Consider z_1 and z_2 be two complex numbers. Let z_1 satisfies $|z_1 - 3 - i| = 2$ and $|z_1 - 2 + i| + |z_1 - 4 - 3i| = 6$imultaneously. Also z_2 is satisfying $|z_2 - 3| \leq |z_2 - 1|, |z_2 - 3| \leq |z_2 - 5|, |z_2 - i| \leq |z_2 + i|$ and $|z_2 - i| \leq |z_2 - 5i|$

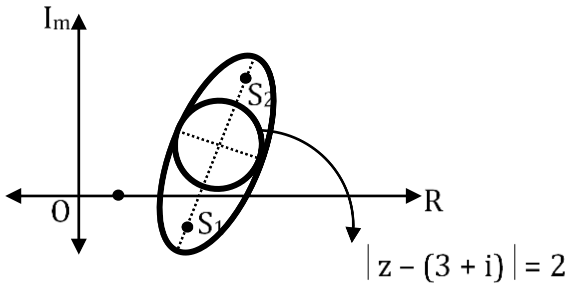
7. Number of possible complex number z_1 is equal to

Ans. **(2)**

Sol. $S_1 = 2 - i\,;\; S_2 = 4 + 3i$

$|z_1 - (2 - i)| + |z - (4 + 3i)| = 6$

$S_1S_2 = 2\sqrt{5}; e = \frac{\sqrt{5}}{3} \Rightarrow b = 2$

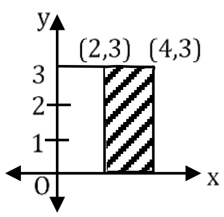


Extremity of minor axis touches the circle ⇒ Two points of contact.

8. The area of region in which z_2 lie is equal to

Ans. **(6)**

Sol.



Area $= 2 \times 3 = 6$

Question Stem-3

Consider the lines L_1 and L_2 defined by

$L_1: x\sqrt{2} + y - 1 = 0$ and $L_2: x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R′ and S′. Let D be the square of the distance between R′ and S′.

9. The value of λ^2 is______.

Ans. **(9)**

Sol. Locus C $= \left|\frac{(x\sqrt{2}+y-1)(x\sqrt{2}-y+1)}{\sqrt{3}}\right| = \lambda^2$

$2x^2 - (y-1)^2 = \pm 3\lambda^2$ for intersection with $y = 2x + 1$

$2x^2 - (2x)^2 = \pm 3\lambda^2$

$-2x^2 = -3\lambda^2$ (taking -ve sign) $\Rightarrow x = \pm\sqrt{\frac{3}{2}}\lambda$

Distance between R and S $= 2\left|\sqrt{\frac{3}{2}}\lambda\right| \sec\theta$ ($\tan\theta$ is slope of line)

$= \sqrt{6}|\lambda|\sqrt{5} = \sqrt{6}|\lambda|\sqrt{5}$

So, $\sqrt{30}|\lambda| = \sqrt{270}(\lambda = \pm 3)$, $\lambda^2 = 9$

10. The value of D is_______.

Ans. **(77.14)**

Sol. Now, the curve C is

$\Rightarrow |2x^2 - (y-1)^2| = 27$

Mid-point of RS is (0,1) and perpendicular bisector is $y = -\frac{1}{2}x + 1 \cdots (2)$

From (1) and (2), we have

$$\frac{7x^2}{2} = 27 \Rightarrow x = \pm 6\sqrt{\frac{3}{7}} \Rightarrow \therefore R'\left(6\sqrt{\frac{3}{7}}, 1 - 3\sqrt{\frac{3}{7}}\right) \text{ and } S'\left(-6\sqrt{\frac{3}{7}} 1 + 3\sqrt{\frac{3}{7}}\right)$$

$$D = (R'S')^2 = \left(12\sqrt{\frac{3}{7}}\right)^2 + \left(6\sqrt{\frac{3}{7}}\right)^2 \Rightarrow D = \frac{3}{7} \times 180 = \frac{540}{7} \Rightarrow \therefore D \approx 77.14$$

SECTION 3

- This section contains **SIX (06)** questions having four options each.
- **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

Full Marks : **+4** If only (all) the correct option(s) is(are) chosen;

Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen;

Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks : **0** If unanswered;

Negative Marks : **−2** In all other cases.

11. Let $S(n) = \sum_{r=1}^{n} \frac{\cos(2.3^{r-1}\alpha)}{\sin(3^r\alpha)}, \alpha = 18^\circ$ arid $P(n) = \sum_{r=1}^{n} \frac{\cos(3.2^{r-1}\beta)}{\cos(2^r\beta)}, \beta = 36^\circ$, then

(A) $S(2012) + P(2012) = 2010$

(B) $S(2012) + P(2012) = 2012$

(C) $P(2011) - S(2011) = 2009$

(D) $P(2011) - S(2011) = 2008$

Ans. **(AD)**

Sol. $2\alpha + 3\alpha = 90^\circ$

$2\alpha = 90^\circ - 3\alpha$

$2.3^{r-1}\alpha = 90^\circ \cdot 3^{r-1} - 3^r\alpha$

$\cos(2.3^{r-1}\alpha) = \pm\sin(3^r\alpha)$

$\Rightarrow \frac{\cos(2.3^{r-1}\alpha)}{\sin(3^r\alpha)} = \begin{cases} 1, & r \text{ is odd} \\ & -1, r \text{ is even} \end{cases}$

Similarly, $\cos(2^r\beta) = \begin{cases} -\cos(3.2^{r-1}\beta), r = 1 \\ \cos(3.2^{r-1}\beta), r \geq 2 \end{cases}$

12. If $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{1}{a^i \cdot a^j} = \frac{\lambda a^2}{(a-1)^2(a+1)}$

where $i \neq j$ and $a > 1$ then 2λ must be greater than

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(ABC)**

Sol. When no restriction on i and j

$$S=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{1}{a^ia^j}=\left(1+\frac{1}{a}+\frac{1}{a^2}+\cdots\ldots\infty\right)^2=\frac{a^2}{(a-1)^2}$$

When $i=j$

$$=\sum_{i=0}^{\infty}\frac{1}{a^{2i}}=1+\frac{1}{a^2}+\frac{1}{a^4}+\cdots\ldots\infty=\left(\frac{a^2}{a^2-1}\right)\Rightarrow\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{1}{a^ia^j}=\frac{a^2}{(a-1)^2}-\frac{a^2}{(a^2-1)}\ i\neq j$$

$$\Rightarrow S=\frac{2a^2}{(a-1)^2(a+1)}\Rightarrow\lambda=2$$

13. If $z_1=5+12i$ and $|z_2|=4$ then

(A) Maximum $|z_1+iz_2|=17$

(B) Minimum $|z_1+(1+i)z_2|=13-4\sqrt{2}$

(C) Minimum $\left|\frac{z_1}{z_2+\frac{4}{z_2}}\right|=\frac{13}{4}$

(D) Maximum $\left|\frac{z_1}{z_2+\frac{4}{z_2}}\right|=\frac{13}{3}$

Ans. **(ABD)**

Sol. (A) $|z_1+iz_2|\leq|z_1|+|z_2|$

(B) $|z_1+(1+i)z_2|\geq||z_1|-|1+i||z_2||$

(C) $\left|z_2+\frac{4}{z_2}\right|\leq|z_2|+\frac{4}{|z_2|}$

(D) $\left|z_2+\frac{4}{z_2}\right|\geq|z_2|-\frac{4}{|z_2|}$

14. Upon $X-Y$ plane, the path defined by the equation $\frac{1}{x^m}+\frac{1}{y^m}+\frac{k}{(x+y)^n}=0$ is

(A) a parabola if $m=-\frac{1}{2},k=-1,n=0$

(B) a hyperbola if $m=1,k=-1,n=0$

(C) a pair of lines if $m=k=n=1$

(D) a pair of lines if $m=k=-1,n=1$

Ans. **(ABCD)**

Sol. (A) $\sqrt{x}+\sqrt{y}=1\ \Rightarrow\ x+y+2\sqrt{xy}=(1-x-y)^2\ x^2-2xy+y^2-2x-2y+1-0$, is a parabola

(B) $\frac{1}{x}+\frac{1}{y}=1\ \Rightarrow\ xy-x-y=0$, is a hyperbola

(C) $\frac{1}{x}+\frac{1}{y}+\frac{1}{x+y}=0\ \Rightarrow\ x^2+3xy+y^2=0$, which is a pair of lines.

(D) $x+y-\frac{1}{x+y}=0\Rightarrow x+y=\pm1$, a pair of lines.

15. The volume of the parallelopiped whose coterminous edges are represented by the vectors $2\vec{b}\times\vec{c}, 3\vec{c}\times\vec{a}$ and $4\vec{a}\times\vec{b}$ where $\vec{a}=(1+\sin\theta)\hat{i}+\cos\theta\hat{j}+\sin 2\theta\hat{k}$, $\vec{b}$ $=\sin\left(\theta+\frac{2\pi}{3}\right)\hat{i}+\cos\left(\theta+\frac{2\pi}{3}\right)\hat{j}+\sin\left(2\theta+\frac{4\pi}{3}\right)\hat{k}$

$\vec{c}=\sin\left(\theta-\frac{2\pi}{3}\right)\hat{i}+\cos\left(\theta-\frac{2\pi}{3}\right)\hat{j}+\sin\left(2\theta-\frac{4\pi}{3}\right)\hat{k}$ is 18 cubic units, then the values of θ, in the interval $\left(0,\frac{\pi}{2}\right)$ is/are

(A) $\frac{\pi}{9}$ (B) $\frac{2\pi}{9}$ (C) $\frac{\pi}{3}$ (D) $\frac{4\pi}{9}$

Ans. **(ABD)**

Sol. volume $=|[2\vec{b}\times\vec{c}, 3\vec{c}\times\vec{a}, 4\vec{a}\times\vec{b}]|=18$

$24[\vec{a}\vec{b}\vec{c}]^2=18 \Rightarrow [|\ \vec{a}\vec{b}\vec{c}\ |]=\frac{\sqrt{3}}{2}$

$$\text{Now, } [\vec{a}\vec{b}\vec{c}]=\begin{vmatrix}(1+\sin\theta) & \cos\theta & \sin 2\theta\\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right)\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$$

Applying $R_1\to R_1+R_2+R_3$ we get after simplification,

$$|[\vec{a}\vec{b}\vec{c}]|=\sqrt{3}|\cos 3\theta|=\frac{\sqrt{3}}{2}\Rightarrow \cos 3\theta=\pm\frac{1}{2}\Rightarrow 3\theta=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3}\Rightarrow\theta=\frac{\pi}{9},\frac{2\pi}{9},\frac{4\pi}{9}$$

16. If the angles between the vectors $\vec{a}$ and $\vec{b}$, $\vec{b}$ and $\vec{c}$, $\vec{c}$ and $\vec{a}$ are respectively $\frac{\pi}{6},\frac{\pi}{4}$ and $\frac{\pi}{3}$, then the angle the vector $\vec{a}$ makes with the plane containing $\vec{b}$ and $\vec{c}$, is

(A) $\cos^{-1}\sqrt{1-\sqrt{2/3}}$

(B) $\cos^{-1}\sqrt{2-\sqrt{3/2}}$

(C) $\cos^{-1}\sqrt{\sqrt{3/2}-1}$

(D) $\cos^{-1}\sqrt{\sqrt{2/3}}$

Ans. **(B)**

Sol. $|\hat{a}\times(\hat{b}\times\hat{c})|^2=|(\hat{a}.\hat{c})\hat{b}-(\hat{a}.\hat{b})\hat{c}|^2$

$\Rightarrow \sin^2(90^\circ-\theta)\sin^2\frac{\pi}{4}=\left|\left(\cos\frac{\pi}{3}\right)\hat{b}-\left(\cos\frac{\pi}{6}\right)\hat{c}\right|^2$

$\Rightarrow \cos^2\theta\times\frac{1}{2}=\frac{1}{4}+\frac{3}{4}-2\times\frac{1}{2}\times\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}=1-\frac{\sqrt{3}}{2\sqrt{2}}\Rightarrow \cos^2\theta=2-\sqrt{3/2}$

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**

Full Marks : **+4** If ONLY the correct integer is entered;

Zero Marks : **0** In all other cases.

17. Let $f: R \to R^{+}$ be a differentiable function satisfying $f'(x) = 2f(x) \forall x \in R$. Also $f(0) = 1$ and $g(x) = f(x) \cdot \cos^2 x$. If n_1 represent number of points of local maxima of $g(x)$ in $[-\pi, \pi]$ and n_2 is the number of points of local minima of $g(x)$ in $[-\pi, \pi]$ and n_3 is the number of points in $[-\pi, \pi]$ where $g(x)$ attains its global minimum value, then find the value of $(n_1 + n_2 + n_3)$

Ans. **8**

Sol. Given $f'(x) = 2f(x)$

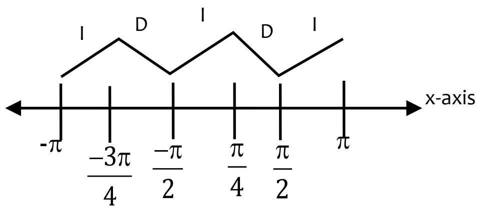
$\therefore \frac{f'}{f} = 2 \Rightarrow f(x) = Ae^{2x}$

$f(0) = 1 = A$

$\therefore f(x) = e^{2x}$

Now, $g(x) = e^{2x} \cdot \cos^2 x$

$g'(x) = e^{2x}(-2\cos x \sin x + 2\cos^2 x)$



$g'(x) = 2 \cdot \cos x \cdot e^{2x}(\cos x - \sin x) = \frac{2e^{2x} \cdot \cos^2 x \cdot (1 - \tan x)}{+ve}$

$G'(x) = 0$

$\Rightarrow x = \frac{-3\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{-\pi}{2}$

$\therefore$ Points of maxima are $\frac{-3\pi}{4}, \frac{\pi}{4}$ and π points of minima are $-\pi, \frac{-\pi}{2}, \frac{\pi}{2}$ and global minimum value occurs at $\frac{\pm\pi}{2}$ which is zero.

Hence $n_1 = 3, n_2 = 3, n_3 = 2$

$\Rightarrow n_1 + n_2 + n_3 = 8$

18. Consider a curve, $x = \begin{cases} \frac{4-y}{2}, & y > 2 \\ (y-2)^{\frac{1}{3}} + 1, & 2 \le y \le 10 \\ \frac{19-y}{3}, & y < 10 \end{cases}$

If area enclosed by the curve, tangent to it at $x = 2$ and the x-axis is $\frac{p}{q}$ where p and q are relatively prime numbers, then find the value of $(p - 19q)$

Ans. **(7)**

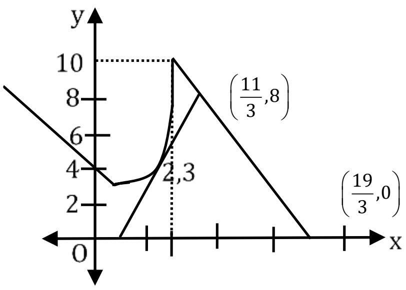
Sol. $y = \begin{cases} 4 - 2x; & x < 1 \\ (x-1)^3 + 2; & [1,3] \\ 19 - 3x; & (3, \infty) \end{cases}$

Tangent at $x = 2$, is $\Rightarrow (y - 3) = 3(x - 2) \Rightarrow 3x - y = 3$

$$\Rightarrow \Delta = \frac{1}{2}\begin{vmatrix} \frac{11}{8} & 8 & 1 \\ 1 & 0 & 1 \\ \frac{19}{3} & 0 & 0 \end{vmatrix}$$

$\Rightarrow \Delta = \frac{64}{3} \equiv \frac{p}{q}$

$\therefore\ p - 19q = 64 - 57 = 7$



19. If the maximum and minimum value of $(\sin x - \cos x - 1)(\sin x + \cos x - 1) \forall x \in R$ is M and m then find value of $(M - 4m)$

Ans. **(6)**

Sol. $y = (\sin x - 1)^2 - \cos^2 x$

$y = (\sin x - 1)^2 - (1 - \sin^2 x)$

$y = 2\left(\left(\sin x - \frac{1}{2}\right)^2 - \frac{1}{4}\right)$

$y_{max} = 4$ when $\sin x = -1$

$y_{min} = -\frac{1}{2}$ when $\sin x = \frac{1}{2}$

$\therefore M = 4; m = -\frac{1}{2}$

PART – 2 : PHYSICS

SECTION 1

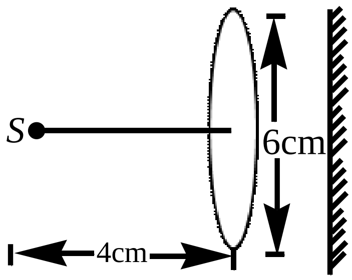
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+3** If ONLY the correct option is chosen;

  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : **−1** In all other cases.

20. In front of a convex lens a light source of power 400 watts is placed at the focus of the lens. Diameter of the lens is 6cm and its focal length is 4cm. 50% of incident light energy is transmitter through a lens and incident normally on a perfectly reflecting surface behind the lens. The force acting on the reflecting surface is 1.33×10^{-n} N. Where 'n' is ________



(A) 6 (B) 7 (C) 8 (D) 3

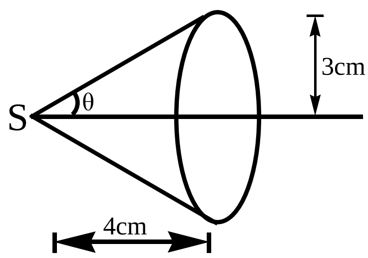
Ans. **B**

Sol. $\tan\theta = \frac{3}{4}$

$\therefore \cos\theta = \frac{4}{5}$

$\therefore$ Solid angle subtended by the lens at the source S is given by

$\Omega = 2\pi(1-\cos\theta) = 2\pi\left(1-\frac{4}{5}\right) = \frac{2\pi}{5} sr$



$\therefore$ Power incident on lens

$P_0 = \frac{400}{4\pi}\Omega = \frac{400}{4\pi}\times\frac{2\pi}{5}$ watt = 40W

Transmitted power $(P) = 0.5 \times 40 = 20W$

The transmitted beam strikes the reflecting surface normally. Momentum incident per unit time on the reflecting surface = $\dfrac{P}{C}$

$\therefore$ Force $= \dfrac{2P}{c} = \dfrac{2 \times 20}{3 \times 10^8} = 1.33 \times 10^{-7}$ N

21. In an H-atom, orbiting electron around proton, radiates energy at the rate of $\dfrac{dE}{dt} = \dfrac{e^2 a^2}{6\pi c^3 \in_0}$, where A is acceleration of the electron, according to the classical mechanics.

Assume that speed of the electron is v (<< c).

Fraction of kinetic energy lost per revolution as a function of velocity v is

(A) $\dfrac{2\pi}{3}\left(\dfrac{v}{c}\right)^3$

(B) $\dfrac{8\pi}{3}\left(\dfrac{v}{c}\right)^3$

(C) $\dfrac{6\pi}{3}\left(\dfrac{v}{c}\right)^3$

(D) $\dfrac{5\pi}{3}\left(\dfrac{v}{c}\right)^3$

Ans. **B**

Sol. We assume that orbit is circular (though the electron will spiral down into the nucleus). If orbital radius at an instant is r then equating the electrostatic force to the centripetal force we get

$$\frac{mv^2}{r} = \frac{1}{4\pi \in_0}\frac{e^2}{r^2} \Rightarrow a = \frac{v^2}{r} = \frac{e^2}{4\pi \in_0 mr} \qquad \text{... (i)}$$

And $\quad v = \sqrt{\dfrac{e^2}{4\pi \in_0 r}}$

Kinetic energy

$$E = \frac{1}{2}mv^2 = \frac{1}{2}.\frac{1}{4\pi \in_0}\frac{e^2}{r} = \frac{e^2}{8\pi \in_0 r} \qquad \text{... (ii)}$$

Time period of circular motion is

$$T = \frac{2\pi r}{v}$$

Energy loss in one revolution is $\Delta E \simeq \left(\dfrac{dE}{dt}\right)T$

$$= \frac{e^2 a^2}{6\pi \in_0 c^3}.\frac{2\pi r}{v} = \frac{e^2 r}{3 \in_0 c^3 v}\left(\frac{v^2}{r}\right)^2 \qquad \left[\because a = \frac{v^2}{r}\right]$$

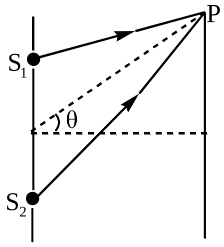
$$= \frac{e^2 v^3}{3\varepsilon_0 c^3 r}$$

$$\frac{\Delta E}{E} = \frac{\Delta E}{\frac{1}{2}mv^2} = \frac{e^2 v^3}{3 \in_0 c^3 r} \times \frac{8\pi \in_0 r}{e^2} \qquad \text{[using (ii)]}$$

$$= \frac{8\pi}{3}\left(\frac{v}{c}\right)^3$$

This means that fraction of energy lost per revolution is small. And because of this reason the radius of the circular path is changing slowly. We can assume the path to be circular for a small interval of time.

22. Two coherent sources emitting light of wavelength λ are $\frac{\lambda}{4}$ apart. I_o is the intensity due to either of the two sources. The intensity at a point in a direction making an angle θ as shown in fig.



(A) $4I_0 \cos^2 \frac{\theta}{2}$ (B) $4I_0 \cos^2 \theta$

(C) $4I_0 \cos^2\left(\frac{\pi}{4}\sin\theta\right)$ (D) $4I_0 \cos^2\left(\frac{\pi}{2}\sin\theta\right)$

Ans. **C**

Sol. Path difference $\Delta = d\sin\theta = \frac{\lambda}{4}\sin\theta$

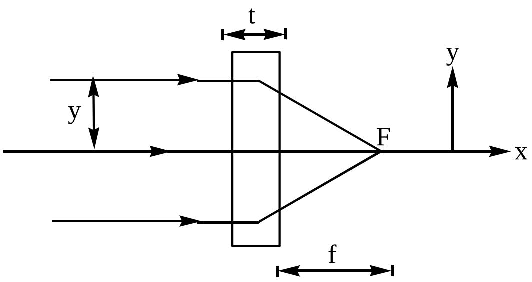
Phase difference $\phi = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}\sin\theta = \frac{\pi}{2}\sin\theta$

$$I_0 = I_1 + I_2 + 2\sqrt{I_1 I_2 \cos\phi} = I_o + I_o + 2\sqrt{I_o . I_o}\cos\left(\frac{\pi}{2}\sin\theta\right)$$

$$= 2I_o + 2I_o\cos\left(\frac{\pi}{2}\sin\theta\right) = 2I_o\left[1+\cos\left(\frac{\pi}{2}\sin\theta\right)\right] = 2I_o \times 2\cos^2\left(\frac{\pi}{4}\sin\theta\right)$$

$$= 4I_0^2\cos^2\left(\frac{\pi}{4}\sin\theta\right)$$

23. A parallel beam of light travelling in x direction is incident on a glass slab of thickness t. The refractive index of the slab changes with y as $\mu = \mu_0\left(1 - \frac{y^2}{y_0^2}\right)$ where μ_0 is the refractive index along x axis and y_0 is a constant. The light beam gets focused at a point F on the x axis. By using the concept of optical path length calculate the focal length f. Assume $f >> t$ and consider y to be small.



(A) $f = \frac{y_0^2}{\mu_0 t}$

(B) $f = \frac{y_0^2}{2\mu_0 t}$

(C) $f = \frac{y_0^2}{10\mu_0 t}$

(D) $f = \frac{5y_0^2}{\mu_0 t}$

Ans. B

Sol. Optical path length of ray along x axis and a ray at a height y will be same.

$$\therefore \quad \mu_0 t + f \simeq \mu t + (y^2 + f^2)^{\frac{1}{2}}$$

$$\left[\mu_0 - \mu_0\left(1 - \frac{y^2}{y_0^2}\right)\right] t = (y^2 + f^2)^{\frac{1}{2}} - f$$

$$\mu_0 \frac{y^2}{y_0^2} t = f\left[\left(1 + \frac{y^2}{f^2}\right)^{1/2} - 1\right]$$

$$\mu_0 \frac{y^2}{y_0^2} t \simeq f \frac{y^2}{2f^2}$$

$$\Rightarrow f = \frac{y_0^2}{2\mu_0 t}$$

SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+2** If ONLY the correct numerical value is entered at the designated place;
  Zero Marks : **0** In all other cases.

Paragraph for Question Nos. 24 & 25

To find index error (e) distance between object needle and pole of the concave mirror is 20 cm. The separation between the indices of object needle and mirror was observed to be 20.2 cm. In some observation, the observed image distance is 20.2 cm and the object distance are 30.2 cm.

24. The index error in cm ________ .

Ans. **D**

Sol. Index error e = observed distance – actual distance

= separation between indices – distance between object needle and pole of the mirror =20.2 – 20.0 = 0.2 cm

25. Assuming equal Index errors for object and image pin, what is the focal length of the mirror in cm ________ .

Ans. **C**

Sol. $|u| = 30.2 - 0.2 = 30cm$

$\therefore u = -30cm$

$|v| = 20.2 - 0.2 = 20cm$

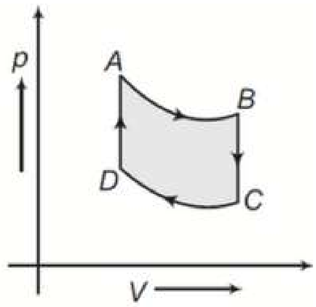
$\therefore v = -20cm$

Using the mirror formula, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{-20} + \frac{1}{-30}$

or $f = -12cm$ Since, it is a concave mirror, therefore focal length is negative

Paragraph for Question Nos. 26 & 27

One mole of a monoatomic ideal gas is taken through the cycle shown in figure.



$A \to B$ Adiabatic expansion

$B \to C$ Cooling at constant volume

$C \to D$ Adiabatic compression

$D \to A$ Heating at constant volume

The pressure and temperature at A, B, etc., are denoted by p_A, T_A; p_B, T_B etc, respectively.

Given, $T_A = 1000K$, $p_B = \left(\frac{2}{3}\right)p_A$ and $p_C = \left(\frac{1}{3}\right)p_A$.

Given, $\left(\frac{2}{3}\right)^{0.4} = 0.85$ and $R = 8.31$ J/mol-K

26. The work done in process $A \to B$ is 373.95n Joule, value of n ________ .

Ans. **D**

Sol. As for adiabatic change $pV^{\gamma} =$ constant

i.e. $P\left(\frac{nRT}{p}\right)^{\gamma} =$ constant (as pV = nRT)

i.e. $\frac{T^{\gamma}}{p^{\gamma-1}} =$ constant so $\left(\frac{T_B}{T_A}\right)^{\gamma} = \left(\frac{p_B}{p_A}\right)^{\gamma-1}$, where $\gamma = \frac{5}{3}$

i.e. $T_B - T_A\left(\frac{2}{3}\right)^{1-\frac{1}{\gamma}} = 1000\left(\frac{2}{3}\right)^{\frac{2}{5}} = 850K$

so, $W_{AB} = \frac{nR[T_F - T_I]}{[1-\gamma]} = \frac{1 \times 8.31[1000 - 850]}{\left[\left(\frac{5}{3}\right) - 1\right]}$

i.e. $W_{AB} = 1869.75J$ c

27. The heat lost by the gas in process $B \rightarrow C$ is 529.76m Joule, value of m ________ .

Ans. **D**

Sol. For $B \rightarrow C$, V = constant so $\Delta W = 0$

So, from first law of the thermodynamics $\Delta Q = \Delta U = \Delta W = nC_V \Delta T + 0$

or $\Delta Q = 1 \times \left(\frac{3}{2}R\right)(T_C - 850)$ $\left(as\ C_V = \frac{3}{2}R\right)$

Now, along path BC, V = constant; $p \propto T$

i.e. $\frac{p_C}{p_B} = \frac{T_C}{T_B}$

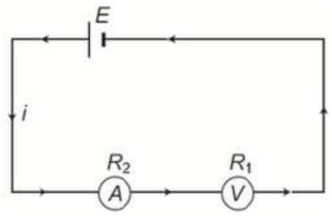
$$T_C = \frac{(1/3)p_A}{(2/3)p_A} \times T_B = \frac{T_B}{2} = \frac{850}{2} = 452K \qquad \text{(ii)}$$

So, $\Delta Q = 1 \times \frac{3}{2} \times 8.31(425 - 850) = -5297.625J$

[Negative heat means, heat is lost by the system]

Paragraph for Question Nos. 28 & 29

A voltmeter of resistance R_1 and an ammeter of resistance R_2 are connected in series across a battery of negligible internal resistance. When a resistance R is connected in parallel to voltmeter, reading of ammeter increases three times while that of voltmeter reduces to one third.

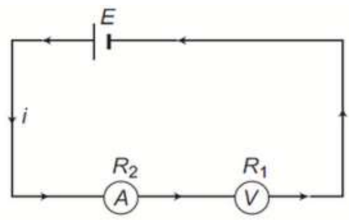


28. $R_1 = nR$, value of n ________ .

Ans. **A**

Sol. Let E be the emf of the battery. In the second case main current increase three times while current through voltmeter will reduce to i/3. Hence, the remaining 3i – i/3 = 8i/3 passes through R as shown in figure.

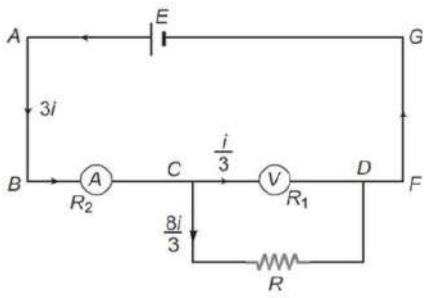
$$V_C - V_D = \left(\frac{i}{3}\right)R_1 = \left(\frac{8i}{3}\right)R \text{ Or } R_1 = 8R$$



29. $R_2 = \dfrac{mR}{3}$, value of m ________ .

Ans. **A**

Sol.



In the second case, main current becomes three times. Therefore, total resistance becomes 1/3 times or $R_2 + \dfrac{RR_1}{R+R_1} = \dfrac{1}{3}(R_1 + R_2)$

Substituting $R_1 = 8R$, we get $R_2 = \dfrac{8R}{3}$

SECTION 3

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+4** If only (all) the correct option(s) is(are) chosen;
- Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;

  Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen, both of which are correct;

  Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;

  Zero Marks **: 0** If unanswered;

  Negative Marks **: −2** In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get + 4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

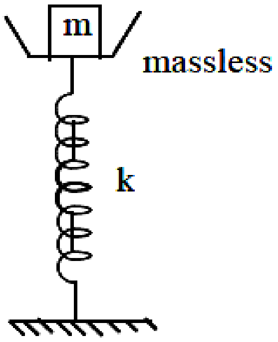
  choosing ONLY (B) will get +1 mark;

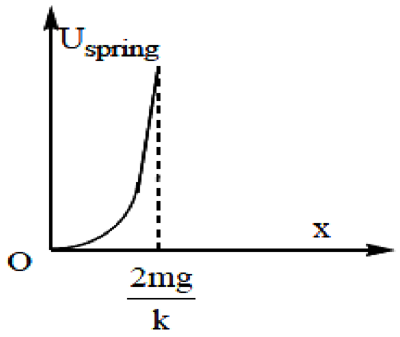
  choosing ONLY (D) will get +1 mark;

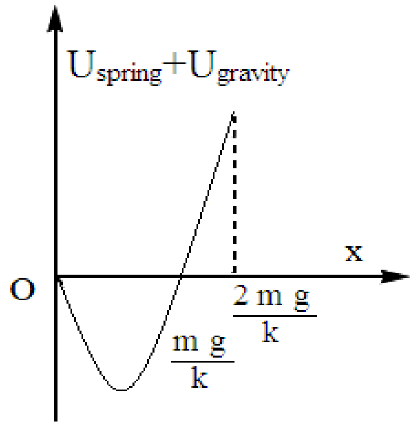
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

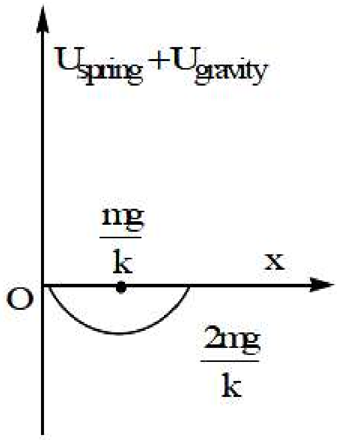
  choosing any other option(s) will get -2 marks.

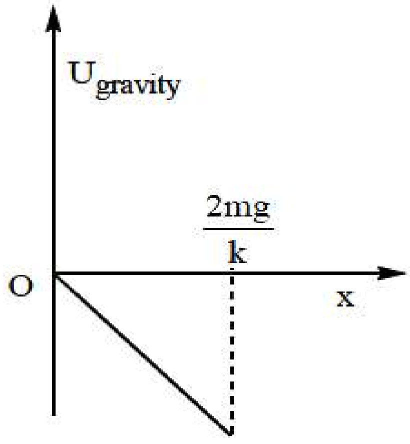
30. A block of mass m is placed on a massless spring (initially unstretched) and released. Due to mass, oscillation starts. There is no loss of energy and initially gravitational potential energy $\left(U_{gravity}\right)$ is assumed to be 0. Potential energy U will change with displacement x of the block as



(A)


(B)


(C)


(D)


Ans. **ACD**

Sol. At equilibrium position $mg = kx \quad \Rightarrow x = \frac{mg}{k}$

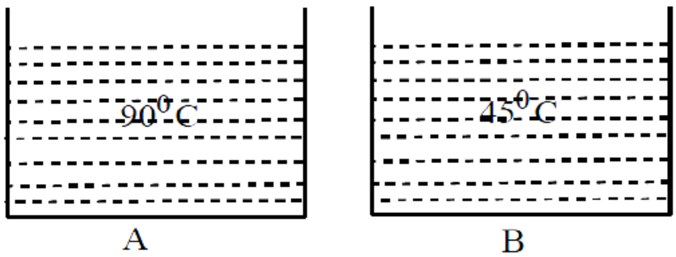
At maximum compression $\frac{1}{2}kx^2 = mgx \Rightarrow x = \frac{2mg}{k}$

$$U_{spring} = \frac{1}{2}kx^2$$

$$U_{gravity} = -mgx$$

$$U_{spring} + U_{gravity} = \frac{1}{2}kx^2 - mgx$$

31. 1 kg water at 90ºC and 1.5 kg water at 45ºC is kept in two identical containers A and B respectively, each of water equivalent 0.5 kg. If water of container A is poured into container B, the final temperature of mixture is T_1, and if water of container B is poured into container A the final temperature is T_2 (heat loss is negligible). Then correct option(s) will be



(A) $T_1 = 60°C$

(B) $T_2 = 60°C$

(C) $T_2 - T_1 = 7.5°C$

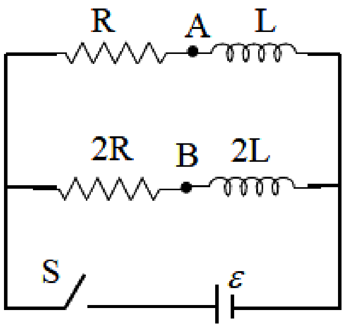
(D) $T_1 = T_2$

Ans. **AC**

Sol. $1C(90 - T_1) = (1.5 + 0.5)C(T_1 - 45) \quad \Rightarrow T_1 = 60°C$

$1.5C(T_2 - 45) = (1 + 0.5)C(90 - T_2) \quad \Rightarrow T_2 = 67.5°C$

32. In the given LR circuit, the switch S is closed at time t = 0. Then,



(A) The ratio of induced emfs in the inductors of inductance L and 2L will be constant

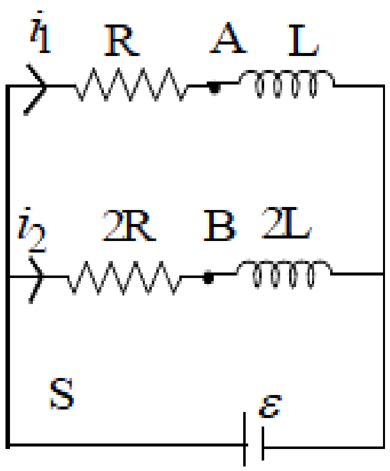
(B) The ratio of induced emf's in the inductor of inductance L and 2L will be time varying

(C) The potential difference '$V_A - V_B$' increases with time

(D) The potential difference '$V_A - V_B$' will be constant.

Ans. **AD**

Sol.



$$I_1 = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

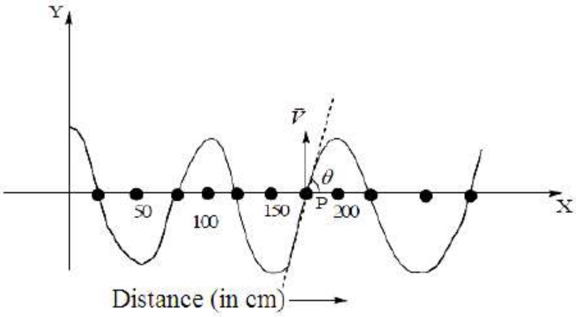
$$\left(\varepsilon_{ind}\right)_L = L\frac{dI_1}{dt} = \varepsilon\left(e^{-Rt/L}\right)$$

$$\left(\varepsilon_{ind}\right)_{2L} = 2L\frac{dI_2}{dt} = \varepsilon\left(e^{-Rt/L}\right)$$

$$V_A - V_B = -2L\frac{dI_2}{dt} + L\frac{dI_1}{dt} = 0$$

33. Shape of string transmitting wave along x-axis at some instant is shown.

Velocity of point P is $v = 4\pi$ cm/s and $\theta = \tan^{-1}(0.004\pi)$



(A) Amplitude of wave is 2 mm

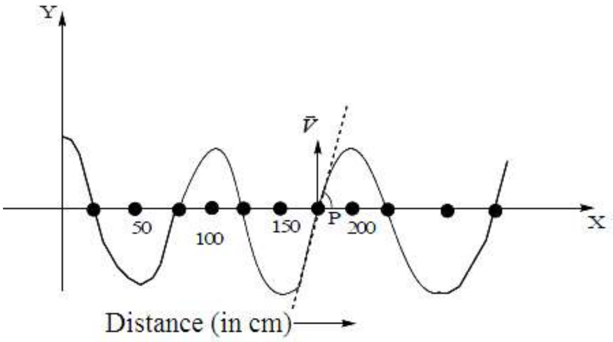
(B) Velocity of wave is 10 m/s

(C) Max acceleration of particle is $80\ \pi^2 \text{cm/sec}^2$

(D) Wave is traveling in – ve x-direction

Ans. **ABCD**

Sol.



$\lambda = 1\text{m}$

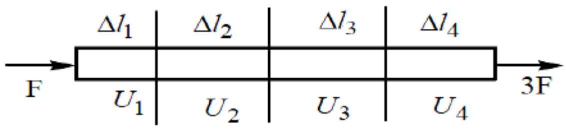
$v_{max} = A\omega = 4\pi$

$$\therefore v = \frac{\partial\lambda/\partial t}{\partial\lambda/\partial x} = \frac{4\pi \times 10^{-2}}{4\pi \times 10^{-3}} = 10\,\text{m/s}$$

$A = 2 \times 10^{-3}\text{m}$

also $v = \dfrac{V}{\lambda} = 10\,\text{Hz}$ and $a_{max} = A\omega^2 = 80\pi\ \text{cm/sec}^2$

34. If a rod of length ℓ, very small area of cross-section A, Young's modulus of elasticity Y is acted upon by two parallel forces 3F and F respectively (as shown) and placed on a smooth horizontal plane. If the elastic limit is not crossed and then to study the change in length of rod $(\Delta\ell)$ and it's elastic potential energy (U) the rod is segmented into four equal parts where magnitude of change in lengths are $\Delta\ell_1, \Delta\ell_2, \Delta\ell_3, \Delta\ell_4$ and elastic potential energy stored in each segment are U_1, U_2, U_3, U_4 respectively as shown then which is/are correct?

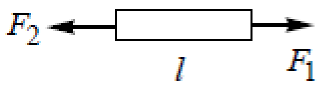


(A) $\Delta\ell_1 = \dfrac{\Delta\ell_3}{3} = \dfrac{\Delta\ell}{8}$ (B) $U_1 < \dfrac{U}{4} < U_4$

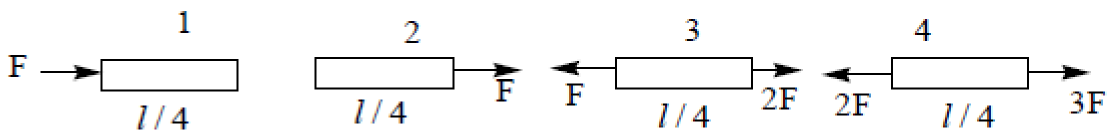
(C) $\Delta\ell_2 = \dfrac{\Delta\ell_4}{5} = \dfrac{\Delta\ell}{8}$ (D) $U_2 < U_3 < U_4$

Ans. **ABCD**

Sol. In steady state, the F.B.D. of the rod can be drawn by dividing it into four equal parts as shown. We have $Y\dfrac{\Delta\ell}{\ell} = (\text{stress})_{\text{average}}$ $\Rightarrow \Delta\ell = \dfrac{\ell}{Y}\cdot\dfrac{(F_1+F_2)}{2A}$



Now, $\Delta\ell_{1(\text{compression})} = \dfrac{F\frac{1}{4}}{2AY} = \dfrac{F\ell}{8AY}; \Delta\ell_{2(\text{elongation})} = \dfrac{F\ell}{8AY}$



$\Delta\ell_{3(\text{elongation})} = \dfrac{3F\ell}{8AY}, \Delta\ell_{4(\text{elongation})} = \dfrac{5F\ell}{8AY}$

$\Delta\ell = \dfrac{8F\ell}{8AY} = \dfrac{F\ell}{AY}, \Delta\ell_{4(\text{elongation})} = \dfrac{5F\ell}{8AY}$

$\Delta\ell = \dfrac{8F\ell}{8AY} = \dfrac{F\ell}{AY}$ $\Delta U \propto \text{strain}^2 \Rightarrow \Delta U \propto (\Delta\ell)^2$

35. The field potential in the certain region of space depends only on the x-coordinate as $V = -ax^3 + b$, where a and b are constant. Choose the incorrect statement/s.

(A) The electric flux through the spherical region of radius R, whose centre lies at origin is zero

(B) The total charge enclosed by the spherical region of radius R, whose centre lies at origin is zero

(C) Field lines in the x-y plane must be represented by straight lines

(D) Equipotential lines in the x-y plane must be parallel to x-axis

Ans. **D**

Sol. $\vec{E}.d\vec{s} = \left(3ax^2\hat{i}\right)\cdot ds\frac{\left(x\hat{i}+y\hat{j}+z\hat{k}\right)}{R} = \frac{3ax^3}{R}ds$

$\Rightarrow \phi = 0; \qquad \Rightarrow q = 0$

Electric field lines are straight line, equipotential lines are parallel to y-axis

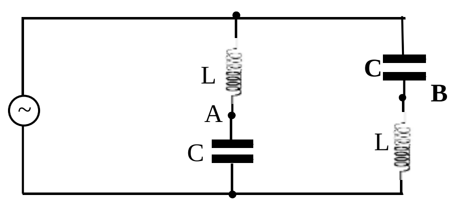
SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+ 4** If **ONLY** the correct integer is entered;

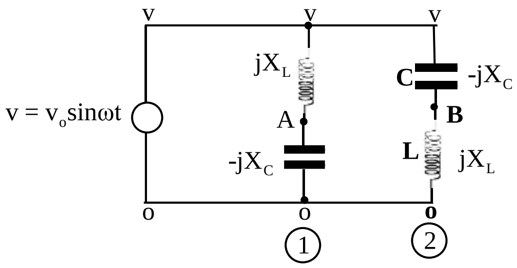
  Zero Marks : **: 0** In all other cases.

36. A circuit shown consists of two identical coils each of inductance L, two identical capacitors each of capacitance C and a variable frequency of the A.C source. Expression of the angular frequency of the source at which the peak voltage between the terminals A and B becomes η times of the peak voltage of the source, for $(X_L > X_C)$ is $\omega = \sqrt{\frac{1+\eta}{\eta-1}\left(\frac{P}{5LC}\right)}$ where P =



Ans. **5**

Sol.



In Branches (1) & (2), Z is same

$$Z = jX_L - jX_C$$

From ckt

For (1) branch

$$V_A = V - \frac{V}{|Z|} jX_L$$

For (2) branch

$$V_B = V - \frac{V}{|Z|}(-jX_C)$$

$$\therefore\ V_B - V_A = V_{BA} = j\frac{V}{|Z|}\left(X_C + X_L\right)$$

$$V_{BA} = \frac{jv_o \sin\omega t}{|Z|}(X_C + X_L)$$

$$V_{BA} = \frac{v_o \sin\left(\omega t + \pi/2\right)}{X_L \sim X_C}\left(X_C + X_L\right)$$

$$\left(V_{BA}\right)_{peak} = \frac{v_o\left(X_C + X_L\right)}{X_L \sim X_C}$$

$$\left(V_{BA}\right)_{peak} = \eta V_o$$

$$\frac{\cancel{v_o}}{X_L \sim X_C}\left(X_C + X_L\right) = \eta \cancel{V_o}$$

$$\boxed{\eta = \frac{X_C + X_L}{X_L \sim X_C}}$$

$$X_L > X_C \rightarrow \eta = \frac{\frac{1}{\omega C} + \omega L}{\omega L - \frac{1}{\omega C}} = \frac{1 + \omega^2 LC}{\omega^2 LC - 1}$$

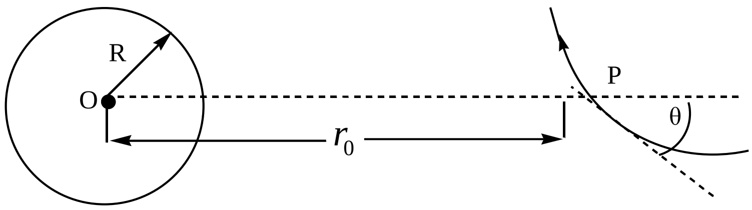
$$\Rightarrow \eta\omega^2 LC - \eta = 1 + \omega^2 LC$$

$$\omega^2 LC\left(\eta - 1\right) = 1 + \eta$$

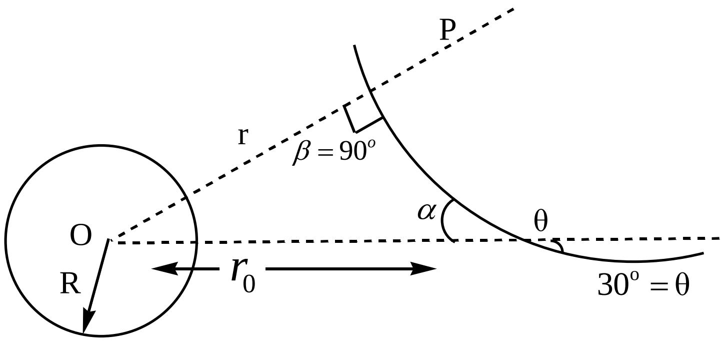
$$\boxed{\omega = \sqrt{\frac{1+\eta}{\eta - 1}\left(\frac{1}{LC}\right)}}$$

37. A laser beam propagates through a spherically symmetric medium surrounding a metal sphere of radius R = 10 cm. Refractive index of the medium varies with distance r from centre O of the sphere according to the law $\mu(r) \propto r$. Here $R << r < \infty$.

The laser beam makes angle of $\theta = 30^{o}$ with a radial line at point P, which is $r_0 = 50\sqrt{2}$ cm away from O. If the minimum distance from surface of the sphere can the beam reach is x cm. (integer) , then the value of $\frac{x}{10}$ will be ?



Ans. 4



Sol.

$\mu = \text{kr}$

Using Snell's law

$$\text{kr}_o \sin\theta = \mu \sin\alpha \longrightarrow (1)$$

ΔOPQ

$$\frac{r_o}{\sin\beta} = \frac{r}{\sin\alpha}$$

$$\sin\alpha = \frac{r}{r_o}\sin\beta \longrightarrow (2)$$

From (1) & (2)

$$\text{kr}_o \sin\theta = \mu\left(\frac{r}{r_o}\right)\sin\beta$$

$$\text{kr}_o \sin\theta = \text{kr}\left(\frac{r}{r_o}\right)\sin\beta$$

$$\text{kr}_o \sin\theta = \frac{\text{kr}^2}{r_o}\sin\beta$$

$r^2 = \frac{r_o^2 \sin\theta}{\sin\beta}$

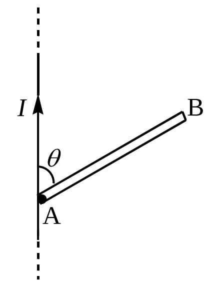
For min approach $\beta = 90^o$

$r = r_o\sqrt{\sin\theta}$

min. distance from the surface

$= r - R = r_0\sqrt{\sin\theta} - R = 50\sqrt{2}\sqrt{\sin 30} - 10 = 40\text{cm}$

38. A conduction rod AB of mass M and length L is hinged at its end A. It can rotate freely in the vertical plane (in the plane of the Figure). A long straight wire is vertical and carrying a current I. The wire passes very close to A. The rod is released from its vertical position of unstable equilibrium. The emf between the ends of the rod when it has rotated through an angle θ (see figure) is $\varepsilon = \frac{\mu_0 I}{2\pi\sin\theta}\sqrt{ngL(1-\cos\theta)}$. Find n



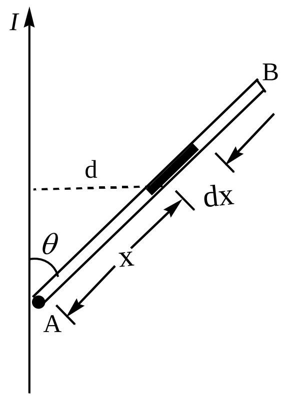
Ans. 3

Sol. We will apply energy conversation to find the angular speed (ω) of the rod.

$\frac{1}{2}I_A\omega^2 =$ loss in gravitational PE

$\frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2 = Mg\frac{L}{2}(1-\cos\theta)$

$\Rightarrow \quad \omega = \sqrt{\frac{3g(I-\cos\theta)}{L}} \quad \text{... (i)}$



Now consider an element of length dx on the rod.

Speed of the element is $v = \omega x$

Magnetic field at the location of the element is $B = \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{2\pi x\sin\theta}$

$\therefore$ Emf induced in the element is $d\varepsilon = Bvdx = \frac{\mu_0 I\omega}{2\pi\sin\theta}dx$

$\therefore$ Emf in the rod is $\varepsilon = \int d\varepsilon = \frac{\mu_0 I\omega}{2\pi\sin\theta}\int_0^L dx = \frac{\mu_0 I\omega L}{2\pi\sin\theta} = \frac{\mu_0 I}{2\pi\sin\theta}\sqrt{3gL(1-\cos\theta)}$

PART – 2 : CHEMISTRY

SECTION 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.

Answer to each question will be evaluated according to the following marking scheme:

- Full Marks : **+3** If **ONLY** the correct option is chosen;

  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : **− 1** In all other cases.

39. For $2A \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} B + 3C$, $2C \xrightarrow{k_2} 3D$, assuming all reactions to be single step (Elementary) reactions, which of the following is **CORRECT**:

(A) $d[C] / dt = 3k_1 [A]^2 - 3 k_{-1} [B] [C]^3 - 2k_2 [C]^2$

(B) $d[B] / dt = k_1 [A]^2$

(C) $d[A] / dt = 2k_{-1} [B] [C]^3 - 2k_1 [B] [C]^3$

(D) $d[B] / dt = 2k_1 [A]^2$

Ans. **A**

Sol. $-\frac{1}{2}\frac{d(A)}{dt} = k_1[A]^2 + k_{-1}[B][C]^3$

40. Addition of dil. HCl to an aqueous solution of a mixture of two inorganic salts yielded white precipitate **E** and filtrate **F**. Precipitate **E** dissolved in hot water. **F** in alkaline alizarin gives a positive red lake test. The cations present in the precipitate **E** and solution **F** respectively are

(A) $Ag^+; Fe^{3+}$ (B) $Hg^{2+}; Ba^{2+}$ (C) $Pb^{2+}; Al^{3+}$ (D) $Pb^{2+}; Zn^{2+}$

Ans. **C**

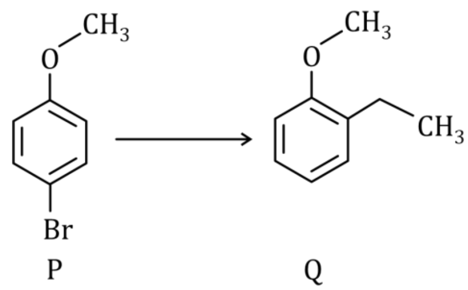
Sol. The aqueous solution of a mixture of two inorganic salts gives white ppt. with dil HCl which dissolves in hot water. It is due to the presence of Pb^{2+}.

$$Pb^{2+}(aq) + 2HCl \longrightarrow \underset{(E)}{\longrightarrow} PbCl_2 \downarrow + 2H^+$$

The filterate (F) is likely to contain Al^{3+} because it gives positive red lake test with alkaline alizarin $Al^{3+} + 3NH_4OH \longrightarrow Al(OH)_3 \downarrow + 3NH_4^+$

gives red lake with alizarine

41. The correct sequence of reactions to get ' Q ' as the only product from ' P ' is



(A) (i) H_2 & Pt catalyst (ii) C_2H_5Cl & $AlCl_3$

(B) (i) Mg in ether (ii) aqueous alcohol (iii) C_2H_5Cl & $AlCl_3$

(C) (i) Mg in ether (ii) C_2H_5Cl & $AlCl_3$

(D) (i) C_2H_5Cl & $AlCl_3$ (ii) Mg in ether (iii) aqueous alcohol

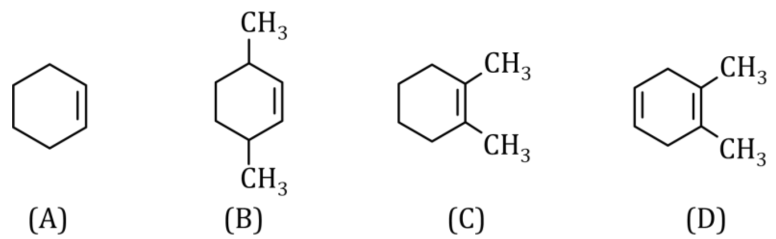
Ans. D

Sol. Alkylation, followed by formation of RMgBr Replacement of RMgBr by H gives the product.

42. Compound ' X ' in the following reaction is

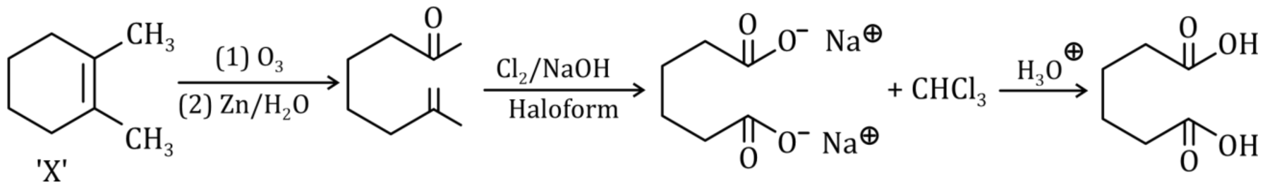
$$X \xrightarrow[\text{(iii) } Cl_2/NaOH \quad \text{(iv) } H_3O^+]{\text{(i) } O_3 \quad \text{(ii) } Zn/H_2O} COOH-((CH_2)_4)-COOH$$

Adipic acid



Ans. C

Sol.



SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
  The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.
  Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : $+$ **2** If **ONLY** the correct numerical value is entered at the designated place;
  Zero Marks : **0** In all other cases.

Paragraph for 43 to 44

At 20ºC the vapour pressure of benzene (C_6H_6) is 75 torr and that of toluene (C_7H_8) is 25 torr. Assume that benzene and toluene form an ideal solution.

43. What is the mole fraction of benzene in solution that has a vapour pressure of 35 torr at 20ºC.

Ans. **0.2**

Sol. $35 = P_T = X_A 75 + X_B \times 25$

44. A solution is prepared by equal moles of benzene and toluene. Find the maximum pressure below which it vapourises completely

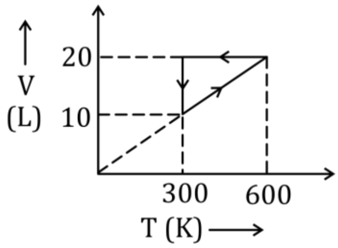
Ans. **37.5**

Sol. $Y_A = \frac{1}{2}$ $\quad$ $Y_B = \frac{1}{2}$

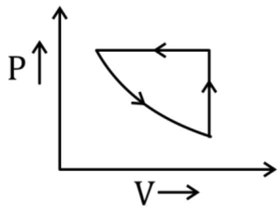
$$\frac{1}{P_T} = \frac{Y_A}{75} + \frac{Y_B}{75}$$

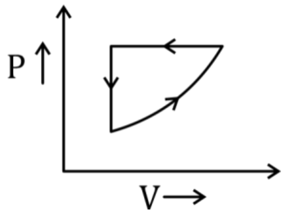
Paragraph for 45 to 46

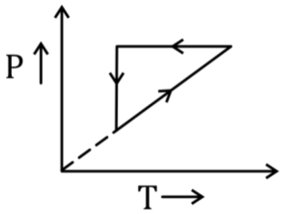
One mol of ideal monoatomic gas undergo the state change as shown in the following graph ($\ln 2 = 0.7$)

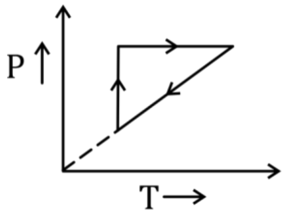


45. No. of incorrect graph for the process in paragraph is –

(A)


(B)


(C)


(D)


Ans. **3**

Sol. AB → Isobaric

BC → Isochoric

AC → Isothermal

46. Efficiency of the cycle will be ________ %.

Ans. **12**

Sol. $q_{abs} = nC_p\Delta T$

$$1 \times \frac{5R}{2} \times 300 = 750\ R$$

$$W_{next} = W_I + W_{II} + W_{III} = -1 \times R \times 300 + 0 + 1 \times 300 \times R \ln 2$$

$$|W| = 300\ R\ (1 - 0.7)$$

$$= 300\ R \times 0.3 = 90\ R = \frac{|W|}{q_{abs.}} \times 100 = \frac{300R \times 0.3}{750} \times 100 = 12\ \%$$

Paragraph for 47 to 48

During an experimental workup procedure, a chemist treated a starting material with NaOH in the solvent acetone $[(CH_3)_2C = O$; however, the starting material was recovered unreacted. Instead, the chemist isolated a small amount of Product A (shown below).

Product A

The chemist determined that Product A resulted from the aldol self-condensation of acetone.

Product A was identified based on the following observations.

Observations about Product A

a. Elemental analysis of Product A indicated that it consisted only of carbon, hydrogen, and oxygen.

b. Product A had a molecular weight of 116 g/mol.

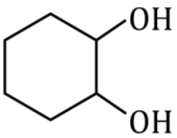
c. Product A was a methyl ketone because it gave a positive iodoform test.

d. When product A was treated with Br_2 in CCl_4, the red bromine colour persisted, because no carbon-carbon double bonds were present to react with the bromine.

The structure of Product A was further confirmed when treatment with hot sulfuric acid resulted in the corresponding dehydration product, Product B.

47. What is the molecular weight of a compound that undergoes an aldol self-condensation reaction to result in a β-hydroxy ketone with a molecular weight of 144 ?

Ans. **72**

Sol. (C)  (mol. mass = 116)

48. Number of compounds from the passage will give a positive iodoform test ?

Ans. **3**

Sol. Product A, Product B and Acetone

SECTION 3

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).

  Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	**+4**	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	**+3**	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	**+2**	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	**+1**	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	**0**	If unanswered;
Negative Marks	:	**−2**	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing ONLY (B) will get + 1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get -2 marks.

49. Which of the following are correct statement

(A) A colloid of $Fe(OH)_3$ is prepared by adding a little excess (required to completely precipitate Fe^{3+} ions as $Fe(OH)_3$)) of NaOH in $FeCl_3$ solution. The particles of this sol will move towards anode during electrophoresis.

(B) During electro-osmosis, colloidal particle moves under influence of electric field

(C) If arsenious sulphide sol is subjected to electrophoresis the colloid particles will move towards anode

(D) When freshly precipitated $Fe(OH)_3$ is shaken with aqueous solution of $FeCl_3$ then on electrophoresis of colloidal solution the colloids particles move towards cathode

Ans. **ACD**

Sol. **Statement 1:** T

Statement 2: F

Statement 3: T

Statement 4: T

50. Select the correct reduction process:

(A) $2[Ag(CN)_2]^- + Zn \longrightarrow [Zn(CN)_4]^{2-} + 2Ag$

(B) $CuO + H_2 \longrightarrow Cu + H_2O$

(C) $Al_2O_3 + 3Zn \longrightarrow 2Al + 3ZnO$

(D) $MgO + C \xrightarrow[\text{temp.}]{\text{high}} Mg + CO$

Ans. **ABCD**

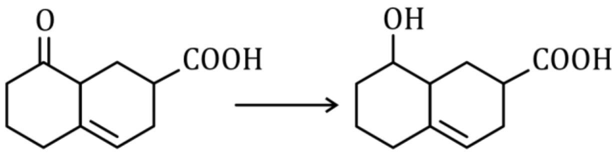
Sol. When an element gives away electrons, it gains charge, or its oxidation number increases. This occurs when an element is oxidized. When an element gains electrons, its oxidation number decreases. This is when an element is reduced. so change in the oxidation numbers of the above reactions:

(A) +1 to 0 (B) +2 to 0

(C) +3 to 0 (D) +2 to 0

In all these cases oxidation number decreased so all are reduction reactions. hence options A,B,C & D are

51. Which one/s of the reduction techniques mentioned below is/are NOT suitable for the following chemical transformation



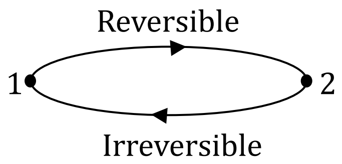
(A) $NaBH_4$ based reduction (B) $LiAlH_4$ based reduction

(C) DIBAL-H based reduction (D) B_2H_6 based reduction

Ans. **BCD**

Sol. $LiAlH_4$ can reduce ketones and carboxylic acid to alcohols. While DIBAL-H can reduce ester, nitriles and acid chlorides to aldehydes. $NaBH_4$ reduces aldehydes, ketones and acid chlorides to alcohols but does not reduce carboxylic acids. B_2H_6 selectively reduces carboxylic acids to alcohols.

52. For the cyclic process given below, which of the following relations are **CORRECT**?



(A) $\Delta S_{1\to2} = S_2 - S_1 = \int_1^2 \frac{\delta q_{rev}}{T}$ (B) $\Delta S_{2\to1} = S_1 - S_2 = \int_2^1 \frac{\delta q_{irr}}{T}$

(C) $\Delta S_{cycle} = \int_1^2 \frac{\delta q_{rev}}{T} + \int_2^1 \frac{\delta q_{irr}}{T} = 0$ (D) $\Delta S_{cycle} = \left(\int_1^2 \frac{\delta q_{rev}}{T} + \int_2^1 \frac{\delta q_{irr}}{T}\right) > 0$

Ans. **AD**

Sol. $dS = \frac{q\ mor}{T}$ and $\oint dS = 0$

53. Four statements for the following reaction are given below

$$[CoCl_2(NH_3)_4]^+ + Cl^- \rightarrow [CoCl_3(NH_3)_3] + NH_3$$

(I) only one isomer is produced if the reactant complex ion is a trans isomer

(II) three isomers are produced if the reactant complex ion is a cis isomer

(III) two isomers are produced if the reactant complex ion is a trans isomer

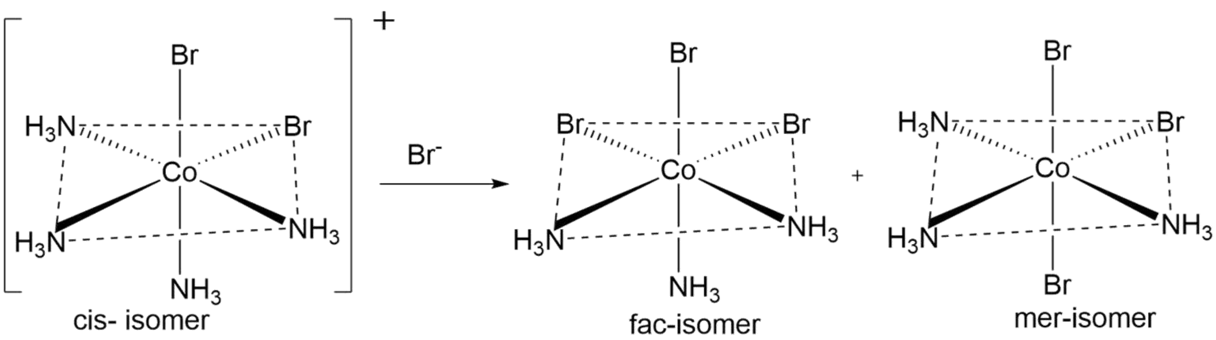
(IV) two isomers are produced if the reactant complex ion is cis isomer

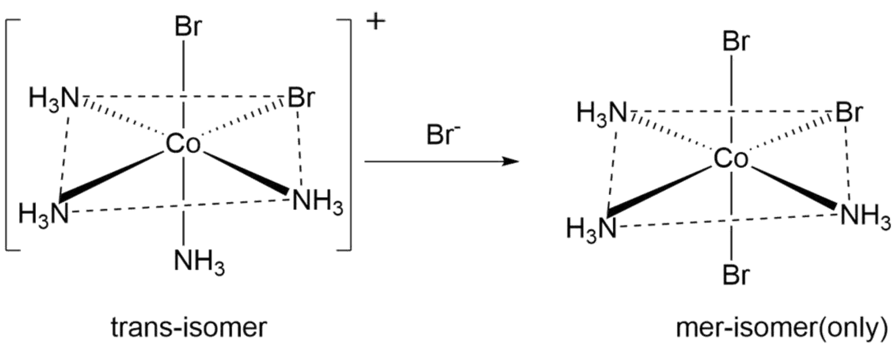
The correct statements are

(A) I (B) III (C) IV (D) II

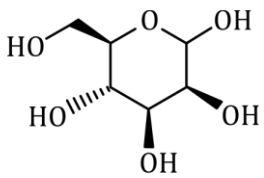
Ans. **AC**

Sol.





54. The correct statement/s about the pyranose form of a sugar (X) given below is/are:



(X)

(A) It exists in two anomeric pyranose forms

(B) It reacts with Tollens' reagent to give a silver mirror

(C) The penta-O-methyl derrvative of (X) is non reducing.

(D) It resists reduction with aqueous sodium borohydride

Ans. **ABC**

Sol. **A.** Correct, it exists in two anomeric forms α and β

B. Correct, as it reduces the Tollens' reagent.

C. Correct, as penta-O-methyl derivative of X can not convert into open chain form of carbohydrates.

So, it is non-reducing.

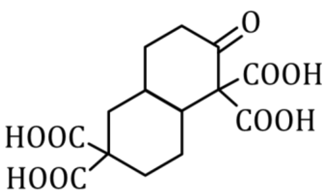
D. Incorrect, it can be reduced by $NaBH_4$

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

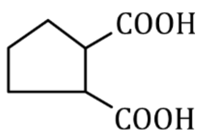
Full Marks	:	**+4**	If **ONLY** the correct integer is entered;
Zero Marks	:	**0**	In all other cases.

55. How many moles of CO_2 per mole will release on heating of following compounds.

(A) 

(B) $CH_2 = CH - CH_2 - \underset{\underset{O}{\|}}{C} - OH$

(C) $Me - \underset{\underset{O}{\|}}{C} - CH_2 - SO_2H$

(D) 

Hint: Give your answer as four digit number, for example - if compound ABCD liberates 1,2,3 and 4 moles of CO_2 respectively on heating fill 1234 in Answer sheet.

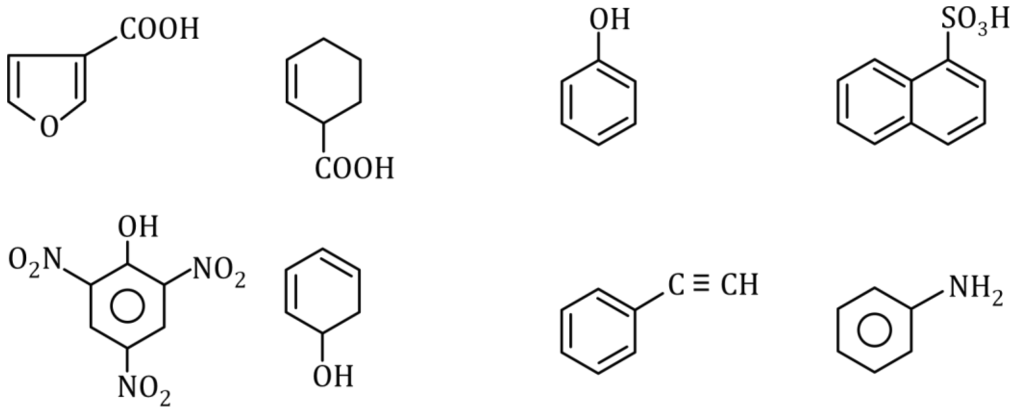
Ans. **3100**

56. In XeF_5^- and XeF_5^+ find the sum of axial d-orbital which are used in hybridization of both species.

Ans. **4**

Sol. XeF_5^-, XeF_5^+ have two $d_{x^2-y^2}$ and d_{x^2} in each participating in the hybridization of orbitals. So, the sum of axial d-orbital which are used in hybridization of both species is 4.

57. Consider following compounds.



(A) Number of compounds which are aromatic (x).

(B) Number of compounds which can evolve H_2 gas on reaction with Na metal (y).

(C) Number of compounds which can evolve CO_2 gas on reaction with $NaHCO_3$ (z).

[If answer is x, y & z respectively, then fill 0xyz in OMR sheet]

Ans. **0684**

Sol. (a) Number of compounds which are aromatic (x). 6

(b) Number of compounds which can evolve H_2 gas on reaction with Na metal (y). 8

(c) Number of compounds which can evolve CO_2 gas on reaction with $NaHCO_3$ (z). 4

[If answer is x, y & z respectively, then fill 0xyz in OMR sheet] 0684